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LETTER

ADDENDUM TO "NONLINEAR VIBRATIONS OF SIMPLY SUPPORTED, CIRCULAR CYLINDRICAL SHELLS, COUPLED TO QUIESCENT FLUID"

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This letter is related to the paper of Amabili *et al.* (1998) and is written for three reasons: (i) to explain better the approximation on tangential boundary conditions used in this paper, (ii) to complete the literature review therein with additional papers, (iii) to correct a few misprints.

1. CORRIGENDA

First of all, let us correct the misprints. The correct form of equations (4), (20a), (68) and (A10) in Amabili *et al.* (1998) is, respectively;

$$(1-\nu^2)\frac{N_x}{Eh} = -\frac{\nu w}{R} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 + \frac{\nu}{2}\left(\frac{\partial w}{R\partial\theta}\right)^2 + \frac{\partial u}{\partial x} + \frac{\nu}{R}\frac{\partial v}{\partial\theta},$$

$$\int_0^{2\pi} N_x R\,\mathrm{d}\theta = 0, \quad \text{(Case 1)},$$

$$w = \{[\tilde{A}_{mn}\cos(\omega t+\vartheta_1) + \tilde{B}_{mn}\sin(\omega t+\vartheta_2)]\cos(n\theta) + \tilde{B}_{mn}\sin(n\theta-\omega t-\vartheta_2)\}\sin(\lambda_m x) + \mathcal{O}(\varepsilon^2),$$

$$c_{10}(t) = \frac{12Ehn^2\lambda_m^2}{2R^2(4\lambda_m^2+n^2/R^2)^2}A_{m0}(t)B_{mn}(t).$$

2. ADDITIONAL LITERATURE

Although the literature review in Amabili *et al.* (1998) is quite extensive, the opportunity is grasped here to make it more complete. In particular, there is some additional work on nonlinear vibrations of infinitely long circular cylindrical shells (rings). Raouf & Nayfeh (1990) and Nayfeh *et al.* (1991) studied the response of the system, retaining both the driven and the companion modes, and found amplitude-modulated and chaotic solutions. The method of multiple scales is applied to obtain a perturbation solution from the equation of motion. In particular, Nayfeh *et al.* (1991) considered the presence of a 2:1 internal resonance. Actually, these papers are only marginally related to the present study, as a consequence of the different and simpler geometry.

Iu & Chia (1988) studied antisymmetrically laminated cross-ply circular cylindrical shells using the Timoshenko–Mindlin kinematic hypothesis, an extension of the Donnell theory of shells. Effects of transverse shear deformation, rotary inertia and geometrical imperfections are included in the analysis. The solution is obtained by the harmonic balance method after Galerkin projection. Only undamped free vibrations are investigated.

Large-amplitude vibrations of thin, circular cylindrical shells with wafer, stringer or ring stiffening have been studied by Andrianov *et al.* (1996) using the Sanders nonlinear shell equations. The solution is obtained using an asymptotic procedure and boundary layer terms to satisfy the shell boundary conditions. Only the trend of the nonlinearity (backbone curve) is obtained; the frequency-response relationship is not investigated.

Popov *et al.* (1998) and Foale *et al.* (1998) investigate different methods to obtain a low-dimensional system from the nonlinear equations of motion of a shallow cylindrical shell panel under periodic axial forcing.

Only two papers on fluid-coupled shells have to be added. The first one is by Engineer & Abrahams (1994), who examined the scattering of sound waves by a baffled circular cylindrical shell of finite length immersed in a light, compressible inviscid fluid. Nonlinearities are attributed only to the shell dynamics, using the model developed by Chu (1961). However, only axisymmetric modes are investigated and the shell is considered to have vanishingly small bending stiffness, i.e., it is assumed to be a cylindrical membrane.

The second one is by Amabili *et al.* (1999) related to the nonlinear stability of a supported circular cylindrical shell *with flow*. A seven-degree-of-freedom model is developed to solve the problem. In particular, two asymmetric modes (driven and companion modes) are taken for both $m = 1$ and 2, where m is the number of longitudinal half-waves. Three axisymmetric modes with an odd number of longitudinal half-waves are added to the four asymmetric modes. Therefore, the model used can be considered an extension of the three-degree-of-freedom one developed by Amabili *et al.* (1998), in which the artificial kinematic constraint between the first and third axisymmetric modes, previously used to reduce the number of degrees of freedom, is removed.

3. ON THE TANGENTIAL BOUNDARY CONDITIONS

In the paper (Amabili *et al.* 1998), the constraints on tangential displacements are satisfied "on the average", yet the continuity of circumferential displacement is satisfied exactly. What is truly meant by those statements, and how it is achieved, was inadequately and insufficiently clearly explained in the original paper; this is a good opportunity for clarifying this issue more comprehensively. In the paper, the following conditions are imposed:

$$\int_0^{2\pi} N_x R\,\mathrm{d}\theta = 0 \quad \text{(Case 1)}, \tag{1}$$

$$\int_0^{2\pi}\int_0^{L} \frac{\partial u}{\partial x}\mathrm{d}x R\,\mathrm{d}\theta = \int_0^{2\pi} [u(L,\theta) - u(0,\theta)] R\,\mathrm{d}\theta = 0, \quad \text{(Case 2)} \tag{2}$$

and for both cases:

$$\int_0^{2\pi}\int_0^{L} \mathrm{N}_{x\theta}\,\mathrm{d}x R\,\mathrm{d}\theta = 0. \tag{3}$$

Equation (1) assures a zero axial force N_x "on the average". The exact condition $N_x = 0$ at $x = 0$ and L requires that

$$\frac{\partial^2 F}{\partial \theta^2} = 0 \quad \text{for } x = 0,\ L \text{ and for any } \theta. \tag{4}$$

Equation(4) can be manipulated to give

$$R^2 \bar{N}_x - n^2\{\cos(n\theta)[c_6(t) + c_7(t) + c_8(t)] + \sin(n\theta)[c_9(t) + c_{10}(t) + c_{11}(t)]\}$$
$$- 4n^2[c_{12}(t)\cos(2n\theta) + c_{13}(t)\sin(2n\theta)] = 0. \tag{5}$$

The condition given by equation (5) cannot be satisfied exactly for any θ; the term on the left-hand side is an oscillating function of main period $2\pi/n$ that has *zero average* on the shell edge. Therefore, the condition $N_x = 0$ at $x = 0$ and L is satisfied on the average, and it is satisfied exactly only at n points on each edge; N_x undergoes oscillations in-between.

Equation (2) states that the axial displacement u is zero "on the average" at $x = 0$ and L. The exact condition $u = 0$ at $x = 0$, L may be transformed into

$$\int_0^L \frac{\partial u}{\partial x} \mathrm{d}x = u(L) - u(0) = 0. \tag{6}$$

Using equations (3a), (4) and (5) of Amabili *et al.* (1998), equation (6) here gives

$$\int_0^L \left[\frac{1}{Eh} \left(\frac{1}{R^2} \frac{\partial^2 F}{\partial \theta^2} - v \frac{\partial^2 F}{\partial x^2} \right) - \frac{1}{2} \left(\frac{\partial w}{\partial x} \right)^2 \right] \mathrm{d}x = 0. \tag{7}$$

After further manipulation, the following expression is obtained:

$$\frac{2L\pi R(-L^2 n^2 + v\pi^2 R^2)[A_{mn}(t)\cos(n\theta) + B_{mn}(t)\sin(n\theta)]}{(L^2 n^2 + \pi^2 R^2)^2} = 0. \tag{8}$$

Equation (8), similar to equation (5) for Case 1, cannot be satisfied exactly for any θ. Analogously, it is satisfied on the average, while being exactly satisfied at n points on each edge.

Equation (3) replaces the exact condition $v = 0$ at $x = 0$ and L that can also be rewritten as

$$\int_0^L \frac{\partial v}{\partial x} \mathrm{d}x = v(L) - v(0) = 0. \tag{9}$$

By using equations (3a) and (6) of Amabili *et al.* (1998), equation (9) can be transformed into

$$-\frac{1}{R} \int_0^L \left[\frac{1 + v}{2Eh} \frac{\partial^2 F}{\partial x \partial \theta} + \frac{\partial w}{\partial x} \frac{\partial w}{\partial \theta} + \frac{\partial u}{\partial \theta} \right] \mathrm{d}x = 0. \tag{10}$$

Unfortunately, the expression of $\partial u/\partial \theta$ cannot be obtained, and equation (10) cannot be expanded further. Therefore, equation (3) is introduced to replace equation (9). Equation (3) can be rewritten as

$$\int_0^{2\pi} \int_0^L \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial \theta} + \frac{\partial w}{\partial x} \frac{\partial w}{\partial \theta} \right) \mathrm{d}x\, \mathrm{d}\theta = 0. \tag{11}$$

In conclusion, equation (9) is satisfied on the average, i.e.

$$\int_0^{2\pi}\int_0^L \frac{\partial v}{\partial x}\,\mathrm{d}x\,\mathrm{d}\theta = \int_0^{2\pi}[v(L)-v(0)]\,\mathrm{d}\theta = 0, \tag{12}$$

when

$$\int_0^{2\pi}\int_0^L \frac{\partial u}{\partial \theta}\,\mathrm{d}x\,\mathrm{d}\theta = \int_0^{L}[u(2\pi)-u(0)]\,\mathrm{d}x = 0 \tag{13}$$

and

$$\int_0^{2\pi}\int_0^L \frac{\partial w}{\partial x}\frac{\partial w}{\partial \theta}\,\mathrm{d}x\,\mathrm{d}\theta = 0. \tag{14}$$

Equation (13) states that u is continuous on the average. Equation (14) is identically satisfied by the mode expansion (7b) used by Amabili *et al.* (1998).

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